**Investments Problem Sheet 9 Lent Term 2024**

**Question 1**

Explain the five major differences between hedge funds and mutual funds.

**Answer:**

The five major categories of differences are transparency, investors, investment strategies, liquidity, and compensation structure.

Mutual funds are more highly regulated by the SEC and thus are required to be far more transparent. Hedge funds provide only minimal information about portfolio composition or strategy.

Investors in hedge funds differ in that investment minimums were traditionally set at $250,000 to $1,000,000. While newer hedge funds are starting to reduce the minimum investment to $25,000, this minimum is outside the reach of many mutual fund investors.

Mutual funds must provide an investment strategy and are restricted in the use of leverage, short selling, and in their use of derivatives. However, hedge funds are less restricted and frequently make large bets that can results in large losses over the short term.

Mutual funds are liquid and investors can redeem shares at NAV and have proceeds within seven business days. Conversely, hedge funds often impose lock-up periods as long as several years and require redemption notices of several months even after the lock-up period is over. Thus, hedge funds are far less liquid.

While mutual funds charge a management fee, hedge funds add an incentive fee as well. This incentive fee is similar to a call option and the portfolio manager receives a "performance" bonus if the portfolio outperforms the chosen benchmark.

**Question 2**

Assuming Paulson & Co charges 20% incentive fee, 0% management fee and the agreed hurdle rate (benchmark rate) is 0%.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year | High-Water Mark | Year End Net Asset Value | Performance Fee |
|  | 2014 | 100 million | 150 million | 10 million |
|  | 2015 | ? | 120 million | ? |
|  | 2016 | ? | 160 million | ? |

Given the above information, compute high-water mark and performance fee in 2015 and 2016.

**Answer:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year | High-Water Mark | Year End Net Asset Value | Performance Fee |
|  | 2014 | 100 million | 150 million | 10 million |
|  | 2015 | 150 million | 120 million | 20% of 0=0 million |
|  | 2016 | 150 million | 160 million | 20% of 10=2 million |

**Question 3**

Consider a passive mutual fund, an active mutual fund, and a hedge fund. The mutual funds claim to deliver the following gross returns:

*rt* passive fund before fees = *rt*stock market index

*rt* active fund before fees = 2.2% + *rt* stock market index + *εt*

The passive fund charges an annual fee of 0.10%. The active mutual fund charges a fee of 1.20% and seeks to beat the same stock market index by about 1% per year after fees. The active mutual fund has a beta of 1 and has a tracking error variance (var(*ε* )) = 3.5%.

The hedge fund uses the same strategy as the active mutual fund to identify “good” and “bad” stocks, but implements the strategy as a long-short hedge fund, applying 4 times leverage. The risk free interest rate is rf=1% and the financing spread is zero (meaning that borrowing and lending rates are equal). Therefore, the hedge fund’s return before fees is

*rt* hedge fund before fees = 1% + 4 × (*rt* active fund before fees − *rt*stock market index )

1. What is the hedge fund’s volatility?

**Answer:**

The hedge fund employs a long-short strategy meaning zero systematic risk with the market and only fund specific risk. Thus, the hedge fund’s volatility is 4×Sqrt(3.5%) = 75%.

1. What is the hedge fund’s beta?

**Answer:**

Since the hedge fund employs a long -short strategy, it indicates that the strategy is market neutral, thus the hedge fund’s beta is zero.

1. What is the hedge fund’s alpha before fees (based on the mutual fund’s alpha estimate)?

**Answer:**

The hedge fund’s alpha before fees is its expected excess return minus its beta times the equity risk premium, i.e., 4×2.20% = 8.80%.

**Question 4.**

You are working for a hedge fund and you have been researching a AAA (ie very low risk of default) 10-year corporate bond which you believe is trading cheaply. You expect the bond to have a return of 7.5% p.a., while the comparable 10-year Treasury bond has an expected return of 6%. The annual volatility of the corporate bond is 6.5% and that of the Treasury bond is 6%; the correlation between them is 0.95. You think of buying the corporate bond and short selling the Treasury to hedge yourself, but the spread of only 1.5% between the expected returns on the two bonds makes this not a very exciting proposition.

Your thoughts turn to leveraging your position. You can borrow or lend short term at the Treasury bill rate of 5.4%.

(a) Supposing that you want to construct a portfolio consisting of the corporate bond, the treasury bond, and borrowing or lending, design the portfolio that has an expected return of 40% and minimum risk.

**If you invest a fraction c of your portfolio in the corporate bond, and a fraction t in the Treasury bond the expected return on the portfolio is:**

**E[*rp*] = 7.5% x *c* + 6% x *t* + 5.4% x (1 – *t – c*) = (5.4 + 2.1*c* + 0.6*t*)%**

**Using the standard formula for the variance of a portfolio, and noting that the risk of T-bills is zero, the variance of the portfolio is:**

**Var[*rp*] = (6.5%)2*c*2 + 2 x 0.95 x 6.5% x 6% x *ct* + (6%)2*t*2**

**= (42.25*c*2 + 74.1*ct* + 36*t*2)%%**

**To get a return of 40%, need:**

**E[*rp*] = 40% so 5.4 + 2.1*c* + 0.6*t* = 40, and**

***t =* (40 – 5.4 – 2.1*c*)/0.6 = 57.667 – 3.5*c*.**

**Substituting for *t* in the variance gives a variance (times 10000) of:**

**Var[*rp*] = 42.25*c*2 + 74.1*c*(57.667 – 3.5*c*) + 36(57.667 – 3.5*c*)2**

**= 223.9*c*2 – 10259*c* +119720**

**Differentiating, this is minimised when *c* = 10259/(2x223.9) = 22.91.**

**So *t* = 57.667 – 3.5*c* = -22.52. Thus for every £1m of initial capital, the portfolio goes long £22.9m of the corporate bond, short £22.5m of the matching ten year bond, and puts £0.6m in cash.**

1. What is the volatility of the portfolio?

**Substituting back, this gives**

**Var[*rp*] =223.9(22.91)2 – 10259(22.91) +119720%%**

**so the standard deviation - the square root of the variance – is 47%. That means that the standard deviation of annual gains or losses relative to the mean is £0.47m on every £1m of capital invested.**

1. You set up the strategy. The correlation turns out to be 0.8 rather than the 0.95 you had assumed. What would you expect the volatility of the portfolio to be?

**Substituting into the formula for volatility gives a portfolio volatility of 90.8%.**

**This example is crudely modelled on one of the strategies followed by LTCM.**

**Question 5.**

Assume that the stock market returns have the market index as a common factor, and that all stocks in the economy have a beta of 1 on the market index. Firm-specific returns all have a standard deviation of 30%. Suppose that an analyst studies 20 stocks, and finds that one-half have an alpha of 2%, and the other half an alpha of -2%. Suppose the analyst buys $1 million of an equally weighted portfolio of positive alphas, and shorts $1 million of an equally weighted portfolio of the negative alpha stocks.

* 1. What is the expected profit (in dollars) and standard deviation of the analyst’s profit?

**Shorting an equally-weighted portfolio of the ten negative-alpha stocks and investing the proceeds in an equally-weighted portfolio of the ten positive-alpha stocks eliminates the market exposure and creates a zero-investment portfolio. Denoting the systematic market factor as RM , the expected dollar return is (noting that the expectation of non-systematic risk, e, is zero):**

**$1,000,000 × [0.02 + (1.0 × RM )] - $1,000,000 × [(–0.02) + (1.0 × RM )]**

**= $1,000,000 × 0.04 = $40,000**

**The sensitivity of the payoff of this portfolio to the market factor is zero because the exposures of the positive alpha and negative alpha stocks cancel out. Thus, the systematic component of total risk is also zero. The variance of the analyst’s profit is not zero, however, since this portfolio is not well diversified.**

**For n = 20 stocks (i.e., long 10 stocks and short 10 stocks) the investor will have a $100,000 position (either long or short) in each stock. Net market exposure is zero, but firm-specific risk has not been fully diversified. The variance of dollar returns from the positions in the 20 stocks is:**

**20 × [(100,000 × 0.30)2] = 18,000,000,000**

**The standard deviation of dollar returns is $134,164.**

1. How does your answer change if the analyst examines 50 stocks instead of 20 stocks? 100 stocks?

**If n = 50 stocks (25 stocks long and 25 stocks short), the investor will have a $40,000 position in each stock, and the variance of dollar returns is:**

**50 × [(40,000 × 0.30)2] = 7,200,000,000**

**The standard deviation of dollar returns is $84,853.**

**Similarly, if n = 100 stocks (50 stocks long and 50 stocks short), the investor will have a $20,000 position in each stock, and the variance of dollar returns is:**

**100 × [(20,000 × 0.30)2] = 3,600,000,000**

**The standard deviation of dollar returns is $60,000.**

**Notice that, when the number of stocks increases by a factor of 5 (i.e., from 20 to 100), standard deviation decreases by a factor of = 2.23607 (from $134,164 to $60,000).**